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Koichiro Sano
Hiroshima University
Yasunobu Tomoda
Kobe City University of Foreign Studies

Institute for Foreign Studies
Kobe City University of Foreign Studies
Nishi-ku, Kobe, 651-2187, Japan

Rat Races in Admission Exam are Beneficial for Society

Koichiro Sano[a], Yasunobu Tomoda[b]
[a] Economics Department, Hiroshima University,
Kagamiyama 1-2-1, Higashihiroshima-shi, 739-8525, Japan
[b] Kobe City University of Foreign Studies,
9-1, Gakuen-Higashimachi, Nishi-ku, Kobe-shi 651-2187, Japan

Abstract

We consider optimal public education policy in a model with a hierarchical education system. While all individuals receive compulsory education, not all of them can access tertiary education due to limited capacity. Individuals who wish to obtain a tertiary education are selected on the basis of performance in school and/or on an admission exam. While severe competition requires students to exert a large effort, it promotes a learning outcome. The optimal education policy consists of the per capita quality of each type of education, the capacity of tertiary education, and the tax rate as a financial resource for public education. We show that the capacity of tertiary education significantly influence competition among students and economic efficiency.

JEL Classification: H52, I21, I28
Keywords: Rat Race, Hierarchical education system, Optimal education policy, Competition among students

1. Introduction

In this paper, we consider optimal public education policy. While public education has multiple functions in an economy, one of its key roles is to provide education for talented children even if they are born into a poor family. Several theoretical studies focus on determining the efficient level of expenditure for public education when innate ability is distributed unevenly.[1] Arrow (1971) concludes that, if different levels of innate ability lead to different marginal productivities in educational investment, the optimal policy is an elite educational system that provides higher education for talented children. Following this result, a large number of studies have analyzed the efficiency and equality of public education for children with different levels of innate ability, frequently under imperfect information, for example, Bruno (1976), Ulph (1977), Maldonado (2008), Oshio and Yasuoka (2009) and Sano and Tomoda (2010).

We believe that the most effective structure for sorting out students with high innate ability is competition within a hierarchical education system. Although all individuals receive compulsory education, only students who are selected through performance-based competition in school and/or on an admission exam go on to advanced education. Through competition, innate abilities are revealed and talented students are given access to advanced education. Moreover, competition encourages students in their learning effort and contributes to human capital formulation. From this perspective, we consider the construction of a suitable hierarchical education system to be a central problem of public education policy.[2]

When considering a hierarchical education system, optimal public education policy has several other critical components. Figure 1.a describes expenditure on educational institutions as a percentage of GDP for all levels of education in 1995, 2000, and 2006. Figure 1.b shows country-specific expenditure ratios of tertiary to non-tertiary (primary, secondary, and post-secondary) education for 2006.[3] These figures lead to several

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[1] The functions of public education include, for example, a response to the imperfection of financial markets for investment in human capital, an internalization of the positive externalities of education, and an equalization of human capital investment and income. These issues are mainly analyzed in dynamic frameworks, e.g., Glomm and Ravikumar (1992), Saint-Paul and Verdier (1993), Eckstein and Zilcha (1994), Benabou (1996), Zhang (1996), Epple and Romano (1998), Benabou (2002), De Fraja (2002), and Cremer and Pestieau (2006).

[2] A few studies set up economic models with hierarchical education systems. Lloyd-Ellis (2000) introduces an occupational choice mechanism that determines which individuals continue on to advance schooling, however, optimal education policy is not considered. Gioacchino and Sabani (2009) show that countries with income inequality choose an elite education system through political process. Oshio and Yasuoka (2009) show how individuals of different abilities decide whether to stay in education or drop out by modeling gradual screening. Arcalean and Schiopu (2010) Sano and Tomoda (2010) study optimal public education policy in hierarchical education system; the central message of this paper is that the desirable distribution of human capital depends on the industrial structure of the country.

[3] Figure 1.b includes both public education expenditure and total education expenditure, which is public and private. Private education complements public education with respect to human capital formation. In particular, Canada, the US, Korea, and Japan have large expenditures on private tertiary education. On the other

questions about optimal education policy: What is the desirable budget size for public education in relation to GDP? How should the government allocate this public education fund between compulsory education and tertiary education? Moreover, what is the suitable capacity for tertiary education? A small capacity and large expenditure per capita in the tertiary education implies an elite education system. On the other hand, a large capacity and small expenditure per capita corresponds to an egalitarian tertiary education.

To answer these questions, we set up a static model with a two-staged hierarchical education system: compulsory education and tertiary education. While individuals who have completed tertiary education become high-skilled workers, individuals who finish schooling with a compulsory education are low-skilled workers. All individuals receive compulsory education, but not all of them can access tertiary education due to a limited capacity. If the number of individuals who wish to enter into tertiary education is larger than the capacity of tertiary education, they are selected based on their scores on an admission exam.[4] The score is an increasing function of their innate ability and study effort. The government collects income tax as the financial resource for public education. The optimal education policy consists of the proportional income tax rate, the expenditures on compulsory and tertiary education, and the capacity of tertiary education.

Assuming homothetic functions for human capital formulation and the production of consumption goods and log-utility, we derive several simple results on the optimal expenditures for compulsory and tertiary educations and the optimal income tax rate. The optimal budget size for public education is decided from the social marginal productivity of educational investment. The government should divide the financial resource according to the marginal productivity of each type of education. On the other hand, we find that the capacity of tertiary education has multiple effects. Firstly, capacity determines the numbers of high-skilled workers. An economy would have the desirable ratio of high-skilled workers to low-skilled workers. Secondly, this influences income inequality. That is, if the government reduces capacity, income inequality expands because high-skilled workers are scarce. Because of log-utility, the increase in income inequality reduces social welfare. Thirdly, the central message of our study is that capacity influences competition among students. The public tertiary educational

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hand, in European countries, public education provides most educational services. While the difference in the share of private education among countries is also an important issue, for simplicity we concentrate our analysis on public education.

[4] Fernandez (1998) and Iyigun Levin (1998) also consider economic models in which individuals are screened by admission exam in higher public education.

institution accepts enrolled students in the order of scores on the admission exam. If tertiary education has a large capacity, highly talented students choose their learning effort level to maximize their life-time utility because a large effort is not required for entry into tertiary education. On the other hand, if the capacity is small, all students set their effort level to survive the competition on the admission exam. This competition requires a large effort from students and causes disutility. The large effort also leads to a high learning outcome, however, which improves the quality of high-skilled workers thereby increasing economic efficiency. Hence, we conclude that a reduction in the capacity of tertiary education has a significant effect in promoting competition among students. Egalitarianism in tertiary education leads to income equality. But, because of this competitive effect, we conclude that egalitarianism is not optimal.

The reminder of this paper is organized as follows. Section 2 describes the model. Section 3 shows market equilibrium and human capital formulation through competition on the admission exam. Section 4 analyzes optimal public education policy. Section 5 summarizes the results.

2. The Model

We set up a static model in order to consider optimal public education policy. The population consists of a unit mass of individuals indexed by $i \in [0,1]$ who are born with heterogeneous innate ability. The innate ability of individuals a_i follows a probability density function $f(a^i)$ with support $a^i \in [a^0, a^1]$, where $\frac{\partial a^i}{\partial i} > 0$ and $0 < a^0 < a^1 < \infty$. Each individual knows the distribution of innate ability and own innate ability. This economy has a hierarchical education system with two types of public educational institutions: compulsory education and tertiary education. We ignore private education for simplification. While individuals who have completed tertiary education become high-skilled workers, individuals who finish their schooling when they have completed compulsory education become low-skilled workers. All individuals receive compulsory education, but not all can access tertiary education due to the limited capacity. If the number of individuals who wish to obtain a tertiary education is larger than the available capacity, they are selected based on their learning outcome, which is typically measured using scores on admission exams. The government collects income tax as the financial resource for public education. The optimal education policy consists of the qualities of compulsory and tertiary education, the capacity of tertiary education, and the proportional income tax rate.

After completing compulsory education, all individuals can become low-skilled workers with human capital z_l, which is formulated by

$$z_l = (E_C)^{\gamma}, \tag{1}$$

where E_C denotes per capita public expenditure on compulsory education and $\gamma \in (0,1)$. Note that this level of human capital is determined independently of innate ability. After compulsory education, each individual decides whether to complete tertiary education or not. The capacity of tertiary education is defined by n, and the tertiary institution accepts enrolled students in the order of scores on the admission exam x^i, which is given by

$$x^i = a^i \cdot e^i, \tag{2}$$

where e^i represents the learning effort level of each individual. Once the have graduated from tertiary education, individuals enter the labor market as high-skilled with human capital

$$z_h^i = (x^i)^{\nu}(E_T)^{\delta}(E_C)^{\eta}, \tag{3}$$

where $\nu \in [0,1)$, $\delta \in (0,1)$, $\eta \in [0,1)$ and $\nu + \delta + \eta \leq 1$. z_h^i and E_T are, respectively, the human capital level of high-skilled workers and per capita public expenditure on tertiary education. Since x^i reflects the learning outcome, the level of human capital depends on innate ability and effort level via x^i.[5]

This economy has one kind of consumption good, which is set as numeraire. The representative firm faces perfect competitive high-skilled and low-skilled labor and consumption goods market. The production function of the firm is given by

$$Y = B(Z_l)^{\beta}(Z_h)^{1-\beta}, \tag{4}$$

where $B > 0$, and $\beta \in (0,1)$. Y denotes output. We assume $\beta\gamma + (\eta + \delta)(1-\beta) < 1$. Z_j ($j = l, h$) are the total amounts of effective labor: $Z_l = \int_{a^0}^{a^1} z_l^i f(a^i)da^i$ and $Z_h = \int_{a^0}^{a^1} z_h^i f(a^i)da^i$.

All individuals have identical preferences

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[5] Liu and Leilson (2011) assume two types effort: real learning and exam preparation, and conclude that the ability to study for the test leads to higher expected test scores but lower skills. We ignore the effort for pure exam preparation, but lower ν implies that learning for exam is not effective for human capital formulation.

$$U = \alpha \log c^i + \psi \log(1 - e^i), \tag{5}$$

where c represents consumption. ψ is an indicator variable: $\psi = 1$ means the individuals go on to tertiary education; and $\psi = 0$ implies they do not. The budget constraint of individual i is

$$(1 - \tau) \cdot I_j^i = c^i, \tag{6}$$

where I_j^i is the wage income of an individual with human capital z_j^i and τ is the income tax rate. Each worker supplies one unit of labor inelastically.

The government provides public education financed by the income tax. The budget constraint of the government is

$$\tau \int_{a^0}^{a^1} I_j^i f(a^i) da^i = E_C + nE_T. \tag{7}$$

The left hand side denotes the tax revenue. The first and second terms on the right-hand side, respectively, represent public expenditures on compulsory and tertiary education.

Our model has the following three stages: In stage 0, the government decides an optimal education policy consisting of E_C, E_T, n, and τ. In stage 1, all individuals receive the compulsory education. After that, some individuals enter tertiary education. In stage 2, individuals work, the representative firm produces goods, and individuals buy and consume them.

3. Market equilibrium and human capital formulation

From the profit maximization of the representative firm, wage rates per unit of effective labor ω_j $(j = l, h)$ are

$$\omega_l = \beta B \left(\frac{Z_h}{Z_l} \right)^{1-\beta}, \tag{8.a}$$

$$\omega_h = (1 - \beta) B \left(\frac{Z_l}{Z_h} \right)^{\beta}. \tag{8.b}$$

Since each worker has human capital z_j^i $(j = l, h)$, the wage incomes of each worker

I_j^i are as follows:

$$I_l = \omega_l z_l \,, \tag{9.a}$$

$$I_h^i = \omega_h z_h^i \,. \tag{9.b}$$

From (5), (6), (9.a), and $\psi = 0$, the indirect utility of low skilled worker is

$$V_{\psi=0} = \alpha \log(1-\tau) I_l = \alpha \log(1-\tau)\omega_l E_C{}^{\gamma} \,. \tag{10.a}$$

On the other hand, from (2), (3), (6), (9.b), and $\psi = 1$, the indirect utility of a high-skilled worker is

$$V_{\psi=1} = \alpha \log(1-\tau) I_h^i + \log(1-e^i)$$

$$= \alpha \log(1-\tau)\omega_h E_T{}^{\delta} E_C{}^{\eta} + \alpha\nu \log a^i + \alpha\nu \log e^i + \log(1-e^i) \,. \tag{10.b}$$

個人の意思決定は賃金を given として解いている。Effort を増やすと賃金も変化するはずであるが、それを考慮せずに効用最大化しているので、社会的に最適になっていないはずである。後で、入試なしで能力の高いほうから n 人を入学させると、努力が過少であることを言う。

Individual i wishes to enter tertiary education if and only if $V_{\psi=0} \le V_{\psi=1}(e^i)$. Figure 2 shows (10.a) and (10.b). Note that $V_{\psi=0}$ is determined independently of e^i. Obviously, (10.b) has a maximum value at $e^i = \dfrac{\alpha\nu}{1+\alpha\nu} \equiv \Lambda$. An individual with a high a^i has a high value for $V_{\psi=1}$. For individuals with $V_{\psi=0} < V_{\psi=1}(e^i = \Lambda)$, $V_{\psi=0}$ and $V_{\psi=1}$ have two intersections which are defined as $\underline{e}^i$ and $\overline{e}^i$:

$$\underline{e}^i < \Lambda < \overline{e}^i \,. \tag{11}$$

Individual i does not go on to tertiary education if the required effort level is greater than $\overline{e}^i$, which depends on a^i. By total differentiation of $V_{\psi=0} = V_{\psi=1}(\overline{e}^i; a^i)$ with respect to $\overline{e}^i$ and a^i, we have

$$\frac{d\overline{e}^i}{da^i} = -\frac{\alpha\nu\overline{e}^i(1-\overline{e}^i)}{a^i[(1+\alpha\nu)\overline{e}^i - \alpha\nu]} > 0 \,. \tag{12}$$

(12) implies that individuals with high innate abilities have large threshold effort levels for admission because innate ability is the marginal productivity of the learning outcome x^i with respect to effort e^i.

We consider the optimal effort level of each individual. The tertiary institution accepts enrolled students in the order of the score (2). Define χ as the lowest score of admission among students accepted by the tertiary institution. The optimal effort level of an applicant for admission is characterized by maximizing (10.b) subject to following two constraints:

$$e^i \leq \bar{e}(a^i), \tag{13}$$

$$x^i = a^i \cdot e^i \geq \chi. \tag{14}$$

Individuals who fail to fulfill (13) do not have an incentive to apply for admission; their effort level is $e^i = 0$. From (12), (13) tightens for individuals with low innate ability a^i. If an individual cannot satisfy (14), the tertiary institution refuses admission. Given χ, (14) also tightens for individuals with low innate ability a^i.

Define $\bar{i} \in i$ as the individual who faces equalities for both (13) and (14), i.e.,

$$e^{\bar{i}} = \bar{e}(a^{\bar{i}}) = \frac{\chi}{a^{\bar{i}}}. \tag{15}$$

It is unique. The optimal effort level of $\bar{i}$ is given by (15). From $\frac{\partial a^i}{\partial i} > 0$, there does not exist e^i which satisfies both (13) and (14) for all individuals $i < \bar{i}$. On the other hand, because all individuals $i > \bar{i}$ can satisfy (14) and (15), they go to the tertiary education. Since $a^{\bar{i}} \cdot \bar{e}(a^{\bar{i}}) = \chi$ from (15), by total differentiation with respect to $\bar{i}$ and χ, we have

$$\frac{\partial \bar{i}}{\partial \chi} = \left[\bar{e}(a^{\bar{i}}) + a^{\bar{i}} \frac{\partial \bar{e}(a^{\bar{i}})}{\partial a^{\bar{i}}}\right]^{-1} \left(\frac{\partial a^{\bar{i}}}{\partial \bar{i}}\right)^{-1} > 0. \tag{16}$$

This implies that a high requirement for the admission exam score imposes a large effort so that individuals with low innate ability decide not to enroll in tertiary education.

Define the total number of applicants for admission of the tertiary education as D; we have

$$D(\chi) = 1 - \bar{i}(\chi), \quad \frac{\partial D}{\partial \chi} = -\frac{\partial \bar{i}}{\partial \chi} < 0. \tag{17}$$

In order to obtain the lowest score for enrollment χ in equilibrium, we consider an imaginary (hypothetical) market for the admission exam. In this market, individuals with higher scores, rather than individuals who pay a higher price, can achieve the entrance qualification. The total demand for tertiary education $D(\chi)$ and the enrollment capacity n decide the lowest score χ, i.e.,

$$D(\chi) = n. \tag{18}$$

(18) determines the lowest score on the admission exam. See figure 3. This means that a high entrance capacity n leads to a low requirement for the admission exam score χ, which corresponds to market price. Summarizing the above discussion, we have the following proposition.

Proposition 1

In the equilibrium for the admission exam, $\bar{i}(\chi) = 1 - n$ holds and all individuals with $i \in [\bar{i}, 1]$ enter tertiary education and all individuals with $i \in [0, \bar{i})$ do not. Moreover, $\frac{\partial \chi}{\partial n} < 0$ and $\frac{\partial \bar{i}}{\partial n} < 0$ are valid.

This is similar to housing market.

Here, we consider the optimal effort level of each individual. It is possible that for some individuals (14) is not binding. Figure 4.a shows the case where (14) is binding; (14) determines the optimal effort level as $e^i = \frac{\chi}{a^i}$. On the other hand, figure 4.b shows the case where (14) is not binding; the optimal effort level is $e^i = \frac{\alpha v}{1 + \alpha v}$.

From (2), the optimal effort levels and scores of individuals are shown in figure 5.a and figure 5.b, where innate abilities are the slopes of the rays. Figure 5.a represents the case where the capacity of tertiary education n is small and competition on the admission exam is severe. In this case, if the individual with the highest innate ability chooses $e^1 = 1$, he or she fails the admission exam. This figure shows that all individuals who go on to tertiary education choose χ. We call this Case 1. From (2), (15), and Proposition 1, the effort levels and scores in Case 1 are, representatively,

$$\begin{cases} e^i = \chi \cdot (a^i)^{-1} = \bar{e}^{1-n} a^{1-n} \cdot (a^i)^{-1} & where \;\; i \in [1-n, 1] \\ e^i = 0 & where \;\; i \in [0, 1-n) \end{cases}, \tag{19}$$

$$\begin{cases} x^i = \chi = \bar{e}^{1-n} a^{1-n} & where \;\; i \in [1-n, 1] \\ x^i = 0 & where \;\; i \in [0, 1-n) \end{cases}. \tag{20}$$

It is similar to Fernandez (1998). On the other hand, figure 5.b shows the case where capacity n is large and the admission exam is not very competitive. We call this Case 2. In this case, because χ is low, individuals with sufficiently-high innate ability can go enroll even when they choose $e^i = 1$. In other words, (14) is not binding for them as in figure 4.b. In Case 2, there exists an individual $\tilde{i}$ $(\bar{i} < \tilde{i} \leq 1)$ who obtains χ by choosing $e^{\tilde{i}} = \dfrac{\alpha v}{1+\alpha v}$. Individuals' effort levels and scores in Case 2 are, respectively,

$$\begin{cases} e^i = \dfrac{\alpha v}{1+\alpha v} & where \;\; i \in [\tilde{i}, 1] \\ e^i = \chi \cdot (a^i)^{-1} = \bar{e}^{1-n} a^{1-n} \cdot (a^i)^{-1} & where \;\; i \in [1-n, \tilde{i}], \\ e^i = 0 & where \;\; i \in [0, 1-n) \end{cases} \tag{21}$$

$$\begin{cases} x^i = a^i \alpha v (1+\alpha v)^{-1} & where \;\; i \in [\tilde{i}, 1] \\ x^i = \chi = \bar{e}^{1-n} a^{1-n} & where \;\; i \in [1-n, \tilde{i}]. \\ x^i = 0 & where \;\; i \in [0, 1-n) \end{cases} \tag{22}$$

An increase in n reduces the requirement score χ. All individuals decrease their effort for entrance into tertiary education, so we have

$$\frac{\partial \tilde{i}}{\partial n} < 0. \tag{23}$$

See figure 5.b. An increase in capacity n raises the numbers of high-skilled workers. However, the policy eases the level of competition on the admission exam. From (2) and (3), this leads to a decrease in the effort level of students and delays human capital formulation for high-skilled workers.

In Case 1, from (19) all graduates of tertiary education achieve the same learning outcome χ and then the same level of human capital. Hence, this economy has identical high-skilled workers and low-skilled workers whose populations are, respectively, n and $1-n$. The total amounts of effective labor are, respectively, $Z_{l,I} = (1-n)z_l = (1-n)E_C^{\ \gamma}$ and $Z_{h,I} = nz_h = \Theta_I E_T^{\ \delta} E_C^{\ \eta}$, where $\Theta_I \equiv n\chi^v$. Subscript I (resp. II) represents Case 1 (resp. Case 2). On the other hand, in Case 2, the amount of low skilled effective labor is also $Z_{l,II} = (1-n)z_l$. From (21), individuals with $i \in [1-n, \tilde{i}]$ formulate the same level of human capital $z_h^i = (\chi)^v E_T^{\ \delta} E_C^{\ \eta}$, while individuals with $i \in [\tilde{i}, 1]$ have different levels of human capital. Therefore, the total amount of high skilled effective labor is $Z_{h,II} = \Theta_{II} E_T^{\delta} E_C^{\eta}$, where $\Theta_{II} \equiv \int_{\tilde{i}}^{1} (a^i)^{\nu} (\Lambda)^{\nu} di + [\tilde{i} - (1-n)]\chi^{\nu}$.

Next, we consider the GDP of the economy and the wage incomes of individuals for each case. From (1), (3), and (4), the total output of consumption goods is

$$Y_k = (1-n)^{\beta} \Theta_k^{1-\beta} B E_C^{\beta\gamma+\eta(1-\beta)} E_T^{\delta(1-\beta)}, \quad k = I, II. \tag{24}$$

From (8.a), (8.b), (9.a), (9.b), and (24), the wage income distribution in Case 1 is

$$\begin{cases} I_{h,I}^{i} = \dfrac{1-\beta}{n} Y_I & where \quad i \in [1-n, 1] \\ I_{l,I}^{i} = \dfrac{\beta}{1-n} Y_I & where \quad i \in [0, 1-n) \end{cases}. \tag{25}$$

In Case 1, because of Cobb-Douglas technology and two identical labor classes, the income from product is distributed in accordance with the marginal productivity of each skilled worker. From (8.a), (8.b), (9.a), (9.b), (22), and (24), we have the wage income distribution in Case 2 as follows:

$$\begin{cases} I_{h,II}^{i} = \dfrac{(1-\beta)a^i\left(\dfrac{\alpha\nu}{1+\alpha\nu}\right)^{\nu}}{\Theta_{II}} Y_{II} & where \quad i \in [\tilde{i}, 1] \\ I_{h,II}^{i} = \dfrac{(1-\beta)\chi^{\nu}}{\Theta_{II}} Y_{II} & where \quad i \in [1-n, \tilde{i}]. \\ I_{l,II}^{i} = \dfrac{\beta}{1-n} Y_{II} & where \quad i \in [0, 1-n) \end{cases} \tag{26}$$

In Case 2, high-skilled workers are separated into upper high skilled with $i \in [\tilde{i}, 1]$ and lower high skilled with $i \in [1-n, \tilde{i}]$. Although the human capital levels of high-skilled workers are not identical, the outcome is also distributed in accordance with the contributions to production of each skilled worker as $\dfrac{\int I_{h,II}^{i} di}{(1-n) I_{l,II}^{i}} = \dfrac{1-\beta}{\beta}$. For guaranteeing $I_l \le I_h^i$, we focus on the policy $n \le 1-\beta$ in Case 1 and the policy $n \le 1 - \dfrac{\beta}{1-\beta} \dfrac{\Theta_{II}}{\chi}$ in Case 2.

We summarize the above results as follows.

Proposition 2

If the government sets a small capacity for tertiary education, all high-skilled workers have identical human capital and wage income. Otherwise, high-skilled workers sort into two classes and income inequality occurs amongst upper skilled workers.

4. Optimal Education Policy

In this section, we consider optimal education policy; in particular, the optimal level of capacity for tertiary education. From (24), (25), (26), the budget constraint of the government (7) is rewritten as

$$\tau \cdot Y = nE_T + E_C. \tag{27}$$

The government chooses E_C, E_T, n, and τ to maximize social welfare

$$W \equiv \int_{a^0}^{a^1} \alpha \log(1-\tau) I_j^i f(a^i) da^i + \int_{a^{1-n}}^{a^1} \log(1-e^i) f(a^i) da^i. \tag{28}$$

subject to (27). Because of the log-utility function and the Cobb-Douglas production function, we can consider the optimal E_C, E_T, and τ exclusive of n. From the first order conditions with respect to E_C, E_T, τ, we have the following proposition.

Proposition 3

In both Case 1 and Case 2, the optimal budget size for public education and the budgetary allocation between compulsory and tertiary education are decided from the following conditions:

$$\tau^* = \beta\gamma + (\eta + \delta)(1-\beta), \tag{29}$$

$$\frac{n^* E_T^*}{E_C^*} = \frac{\delta(1-\beta)}{\beta\gamma + \eta(1-\beta)}, \tag{30}$$

$$\frac{E_C^*}{Y^*} = \{\beta\gamma + \eta(1-\beta)\}, \tag{31}$$

where superscript * represents the optimal solution.

Proof: See Appendix 1.

The implications of these conditions are straightforward. Since we consider a proportional income tax, and all firm revenue is distributed to labor as wages, (29) decides the government's budget size for public education. (30) indicates the optimal ratio of public expenditure for compulsory and tertiary education; it should equal to the ratio of the marginal productivities associated with each type of education. The right hand side of (31) is the share of compulsory education in GDP. From the homothetic production functions for human capital and consumption goods, this share is constant. From these conditions, the financial decision about public education is not difficult for the government. At first, the government should decide the income tax rate as (29), which is the sum of the marginal productivities of each type of education. Next, the government divides the financial resource according to the marginal productivities of each type of education as (30).

The remaining decision of the government is only the optimal capacity for tertiary education n^*. Should the government adopt an elite tertiary education with small capacity n and large educational expenditure per capita E_T, or an egalitarian tertiary education with large capacity and small expenditure per capita? The former corresponds to an elite education system, while the latter implies an egalitarian education system. The answer is given by the first order condition of n. However, the condition is different for Case 1 and Case 2. So, at first, we must show the following lemma.

Lemma 1

This economy has a unique optimal capacity for tertiary education n^*.

Proof: See Appendix 2.

Although the first order condition with respect to n varies by each case, this lemma guarantees the uniqueness of the solution. Because, from (23), $\tilde{i}$ depends on n, we define $\tilde{n}$ that corresponds to $\tilde{i}=1$. Therefore, $n^* \leq \tilde{n}$ (resp. $n^* > \tilde{n}$) leads to Case 1 (resp. Case 2).

If $n^* \leq \tilde{n}$, the optimal capacity must satisfy the following equation:

$$\alpha \log \frac{1-\beta}{\beta}\frac{1-n^*}{n^*} + \frac{\alpha}{(1-\tau^*)n^*}\left[\frac{1-\beta-n^*}{1-n^*} - \nu(1-\beta)\varepsilon_\chi - \delta(1-\beta)\right]$$

$$+\left[\frac{\partial \chi}{\partial n}\int_{a^{1-n^*}}^{a^l} \frac{-1}{a^l-\chi} f(a^l)da^l - \log(1-e^{1-n^*})f(a^{1-n^*})\frac{\partial a^{1-n^*}}{\partial n}\right] = 0, \qquad (32)$$

where $\varepsilon_\chi \equiv -\frac{\partial \chi}{\partial n}\frac{n}{\chi}$ is the capacity elasticity of the educational outcome. The first term in (32) represents the equalization effect; with log-utility, income equality improves welfare and its term is zero when $n = 1-\beta^*$. From (25), this implies no income inequality. The second term includes two effects. An increase in n raises the number of high skilled workers. If the government selects too little capacity, the economy has a small number of high-skilled workers. But, at the same time, the increase in n eases the competition on the admission exam and reduces the learning outcome of high-skilled workers by Proposition 1. The latter effect is harmful for economic welfare. The third term represents the disutility from the learning effort. The increase in n raises the number of students enrolled in tertiary education, but relaxes competition and reduces all students' effort in learning.

If $n^* > \tilde{n}$ in Case 2, the first order condition with respect to n is the following:

$$\log\frac{1-\beta}{\beta}\frac{(1-n^*)}{\Theta_{II}} + \frac{\alpha v}{n^*}\left[n^* \log\chi - \{\tilde{i} - (1-n)\}\varepsilon_\chi\right]$$

$$+\frac{\alpha}{1-\tau}\left[1-\tau-\frac{\beta}{1-\beta} - \delta(1-\beta) + \{-n(1-\tau)+1-\beta\}\frac{\chi}{\Theta_{II}}\right]$$

$$+\left[-\frac{\partial\chi}{\partial n}\int_{a^{1-n}}^{\tilde{i}}\frac{1}{a^i-\chi}f(a^i)da^i - \log\left(1-\frac{1}{a^{1-n}}\right)f(a^{1-n})\frac{\partial a^i}{\partial n}\right] = 0 \qquad (33)$$

where $\Lambda \equiv 1 + \log\Theta_{II} - \frac{n^*}{\Theta_{II}}\frac{\partial\Theta_{II}}{\partial n} + \frac{\partial\tilde{i}}{\partial n}\log\frac{\chi}{a^{\tilde{i}}} - [\tilde{i} - (1-n^*)]\frac{\varepsilon_\chi}{n^*}$. Although (32) is more complex than (32), the three terms in (33) corresponds to the terms of (32).

A simple benchmark case is to equalize incomes between low-skilled and (lower) high-skilled workers. From (25) and (26), the policy is given by $n = 1-\beta$ in Case 1 and $n = 1 - \frac{\beta}{1-\beta}\frac{\Theta_{II}}{\chi}$ in Case 2. To verify that this is not optimal, we have the following proposition.

Proposition 4

The optimal capacity of tertiary education n^* is smaller than the level of capacity that leads to income equality between low-skilled and (lower) high-skilled workers.

Proof: See Appendix 3.

Under the income equalization policy, individuals who wish to go on to tertiary education have no incentive to make the required effort. Proposition 4 claims that this is never optimal. Competition on admission exam requires students to exert a large effort. At the same time, this competition leads to a large learning outcome and improves the quality of high-skilled workers. The improvement of high-skilled workers increases not only the productivity of high-skilled workers but also the marginal productivity of low-skilled workers and their wage incomes. From Proposition 1, a small capacity for tertiary education imposes a high effort on students. This policy promotes the human capital formulation of high-skilled workers. But, from Proposition 3, the policy for promoting competition does not require a larger financial budget; the government just decreases the capacity of tertiary education and increases per capita public expenditure for tertiary education. We conclude that this moderate elitism is the optimal education policy.

5. Conclusions

We have analyzed optimal education policy in a simple model with a hieratical public education system. The optimal income tax rate is decided by the social marginal productivity of educational investment. The government should divide financial resources according to the marginal productivities of compulsory and tertiary education. On the other hand, we have found that a change in the capacity of tertiary education has multiple effects. Especially, capacity influences the intensity of competition on the admission exam. To survive this competition, individuals exert larger effort; this competition improves economic efficiency.

In most of the literature on optimal education policy, competition among students does not appear to have a crucial role. In the real world, however, it is supposed that people choose their learning effort rationally given the economic environment. If we consider that a desirable education brings out the learning effort of students, the optimal education policy should select the intensity of competition among students through a control of the capacity of tertiary education.

At the end of this paper, we refer to remaining issues for the future. First, in the equilibrium on the completion of the admission exam, most students achieve the same score. But, in reality, the scores are distributed due to uncertainty. Individuals with risk-averse attitudes would make strong efforts in learning. Thus, it is important to investigate how uncertainty affects learning outcomes and human capital formulation. Second, while we have considered a two-staged hierarchical education system, it is possible to extend to the model to include a multi-staged education system. Moreover, we have analyzed identical tertiary educational institutions, but, in reality, there are many kinds of tertiary educations that set different levels of difficulty for admission exams and different qualities for education, for example, Epple, Romano and Sieg

(2006). An analysis of the effect of competition among students of the arrangement of tertiary educational institutions is a remaining issue for study.

Appendix 1: Proof of Proposition 3

From (19), (21), (24), (25) and (26), the social welfare is replaced with the following:

$$W_k = \log(1-\tau) + (1-n)\log\beta + n\log(1-\beta) + (1-n)\log(1-n)$$
$$+\Gamma_k + \log Y_k - \Omega_k, \quad k = I, II \quad (A.1)$$

where $\Gamma_I \equiv n\log(1-n)$, $\Gamma_{II} \equiv n\log\Theta_{II} + \int_{\tilde{i}}^{1}\log a^i di + [\tilde{i} - (1-n)]\log\chi$, $\Omega_I \equiv \int_{1-n}^{1}\frac{\chi}{a^i}di$,

and $\Omega_{II} \equiv [1-\tilde{i}] + \int_{1-n}^{\tilde{i}}\frac{\chi}{a^i}di$. The Lagrangian of the government problem is

$$L_k = \log(1-\tau) + (1-n)\log\beta + n\log(1-\beta) + (1-n)\log(1-n)$$
$$+\Gamma_k + \log Y_k - \Omega_k + \lambda[\tau Y_k - E_C - nE_T]. \quad (A.2)$$

Using $\frac{\partial Y_k}{\partial E_C} = [\beta\gamma + \eta(1-\beta)]\frac{Y_k}{E_C}$ and $\frac{\partial Y_k}{\partial E_T} = \delta(1-\beta)\frac{Y_k}{E_C}$, the first order conditions with respect to τ, E_C, and E_T are, respectively,

$$\lambda = \frac{1}{(1-\tau)Y_k}, \quad (A.3)$$

$$\frac{\beta\gamma + \eta(1-\beta)}{E_C} = \lambda\left[1 - \tau[\beta\gamma + \eta(1-\beta)]\frac{Y_k}{E_C}\right], \quad (A.4)$$

$$\frac{\delta(1-\beta)}{E_T} = \lambda\left[n - \tau\delta(1-\beta)\frac{Y_k}{E_T}\right]. \quad (A.5)$$

From (A.3), (A.4), and (A.5), we have (29), (30), and (31). Q.E.D.

Appendix 2: Proof of Lemma 1.

We define $\tilde{n} \in n$ that corresponds to $\tilde{i} = 1$. From (23), $n \le \tilde{n}$ (resp. $n < \tilde{n}$) leads to Case 1 (resp. Case 2). When $n = \tilde{n}$, i.e., $\tilde{i} = 1$, we have $\Theta_I|_{n=\tilde{n}} = \Theta_{II}|_{n=\tilde{n}}$, $\Gamma_I|_{n=\tilde{n}} = \Gamma_{II}|_{n=\tilde{n}}$, and $\Omega_I|_{n=\tilde{n}} = \Omega_{II}|_{n=\tilde{n}}$. Moreover, $\left.\frac{\partial\Theta_I}{\partial n}\right|_{n=\tilde{n}} = \left.\frac{\partial\Theta_{II}}{\partial n}\right|_{n=\tilde{n}} = \chi(1-\varepsilon_\chi)$, $\left.\frac{\partial\Gamma_I}{\partial n}\right|_{n=\tilde{n}} = \left.\frac{\partial\Gamma_{II}}{\partial n}\right|_{n=\tilde{n}} = -\log\tilde{n} - 1$, and $\left.\frac{\partial\Omega_I}{\partial n}\right|_{n=\tilde{n}} = \left.\frac{\partial\Omega_{II}}{\partial n}\right|_{n=\tilde{n}} = e^{1-\tilde{n}} - \frac{\partial\chi}{\partial n}\int_{1-\tilde{n}}^{1}\frac{1}{a^i}di$ hold.

Thus, from (A.2), we find $L_I\big|_{n=\tilde{n}} = L_{II}\big|_{n=\tilde{n}}$ and $\left.\frac{\partial L_I}{\partial n}\right|_{n=\tilde{n}} = \left.\frac{\partial L_{II}}{\partial n}\right|_{n=\tilde{n}}$, i.e., the Lagrangian is continuous and smooth. Each L_k is single peaked on n. Therefore, the government's problem has a unique solution n^*. If $\tilde{n} \geq n^*$ (resp. $\tilde{n} < n^*$), the optimal solution is in Case 1 (resp. Case 2). Q.E.D.

Appendix 3: Proof of Proposition 4

By the definition of $\bar{e}^{1-n}$ in (11) and from (10.a) and (10.b), we have $\bar{e}^{1-n} = \log \frac{I_h^{1-n}}{I_l^{1-n}}$. Thus, the policy of income equality between low skilled and (lower) high-skilled workers implies $\bar{e}^{1-n} = 0$, which leads to $\chi = 0$ and then $\varepsilon_\chi = \infty$. From (31) and (32), we have

$\lim_{n \to 1-\beta} \frac{\partial L_I}{\partial n} = \lim_{n \to 1-\frac{\beta}{1-\beta}\frac{\Theta_{II}}{\chi}} \frac{\partial L_{II}}{\partial n} = -\infty$. Since this Lagrangian is single peaked, we have

$n^* < 1-\beta$ and $n^* < 1 - \frac{\beta}{1-\beta}\frac{\Theta_{II}}{\chi}$. Q.E.D.

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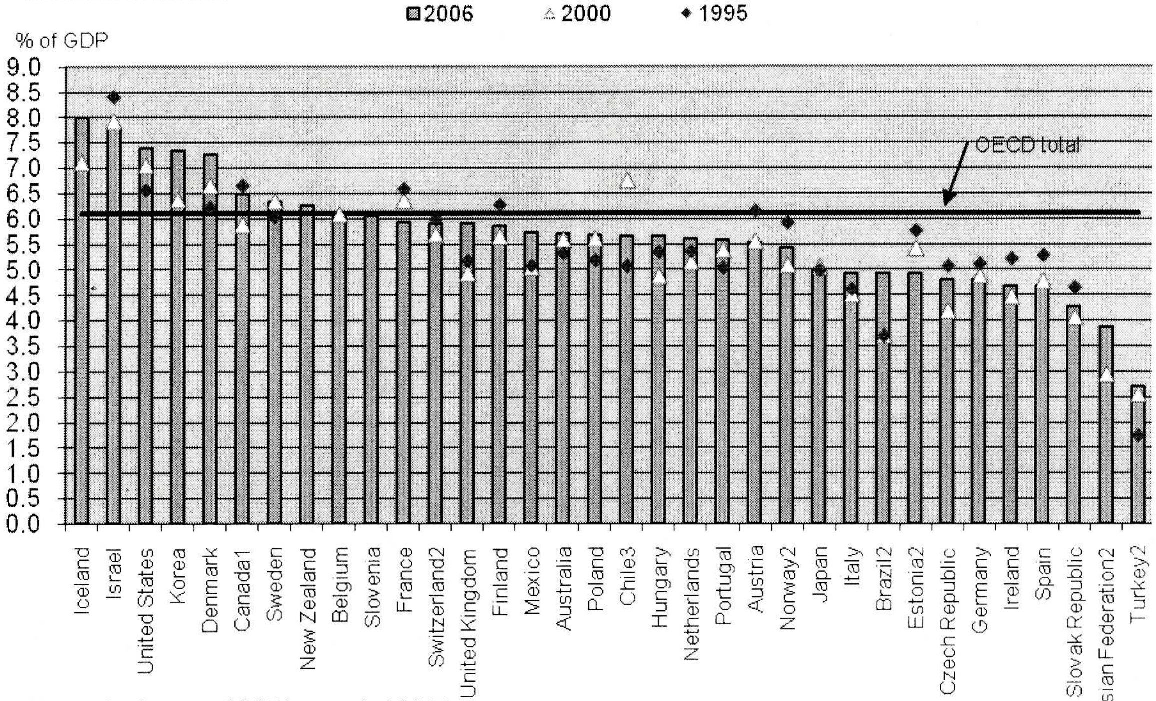
Figure 1.a : Expenditure on educational institutions as a percentage of GDP for all levels of education (1995, 2000, 2006)

This chart measures educational investment through the share of national income that each country devoted to spending on educational institutions in 1995, 2000 and 2006. It captures both direct and indirect expenditure on educational institutions from both public and private sources of funds.



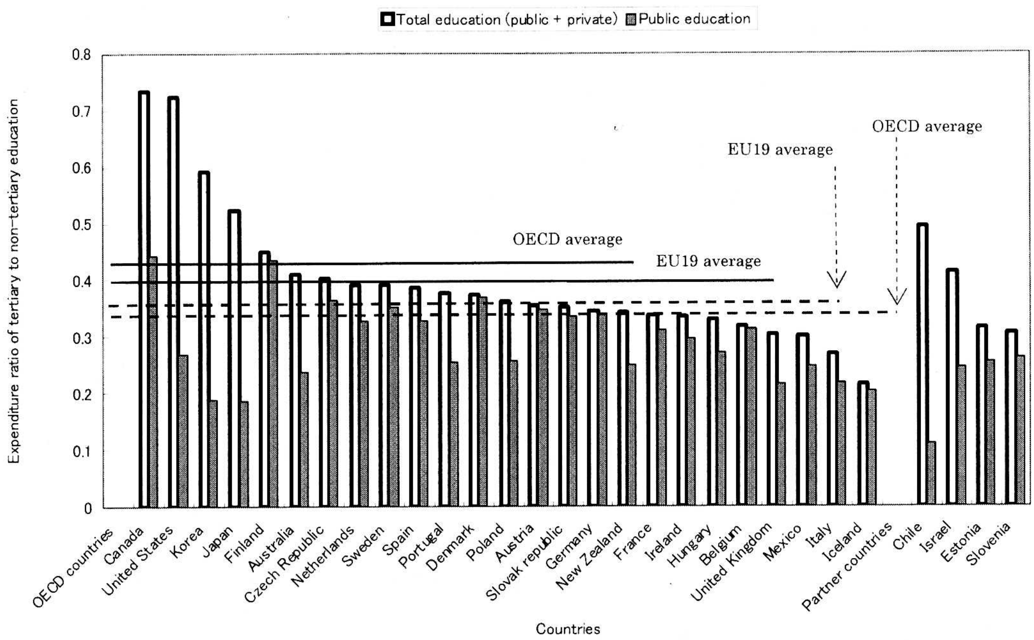
1. Year of reference 2005 instead of 2006.
2. Public expenditure only (for Switzerland, in tertiary education only).
3. Year of reference 2007 instead of 2006.

Countries are ranked in descending order of expenditure from both public and private sources on educational institutions in 2006.

Source: OECD. Table B2.1. See Annex 3 for notes (*www.oecd.org/edu/eag2009*).

Quote from Chart B2.1. OECD 2008. Education at a glance.

Figure 1.b: Country-specific expenditure ratio of tertiary to non-tertiary education



Source: OECD, 2008. Education at a glance.

Figure 2: Indirect utility functions

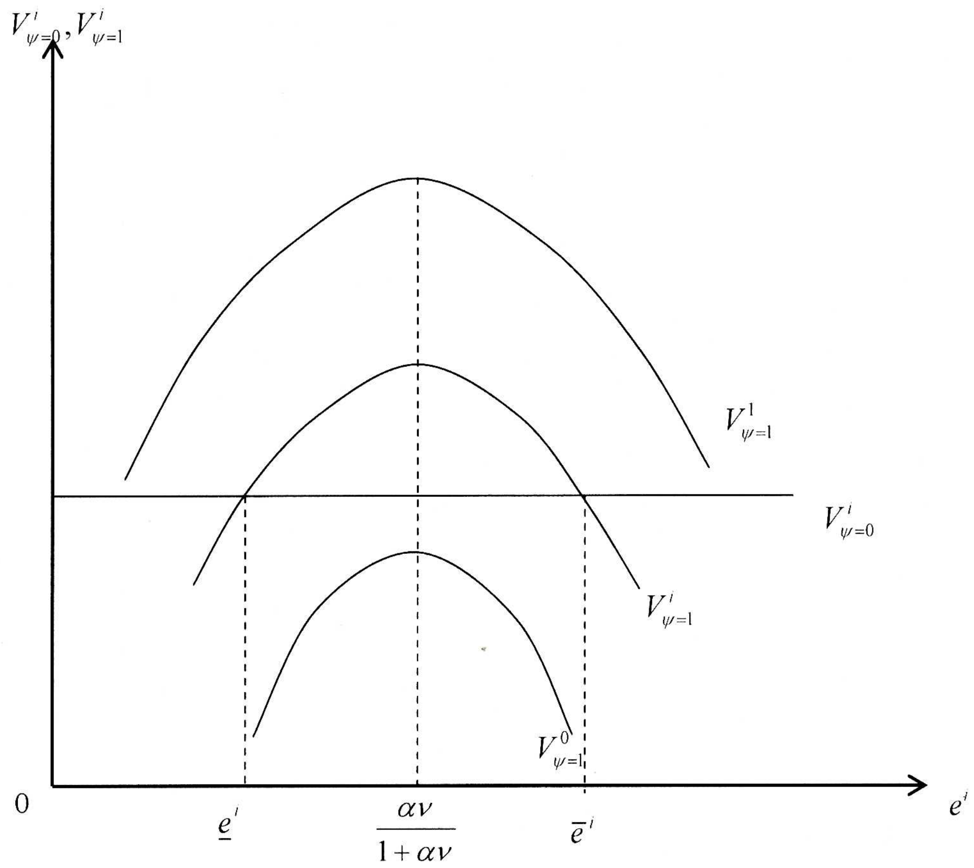


Figure 3: An imaginary market of admission exam

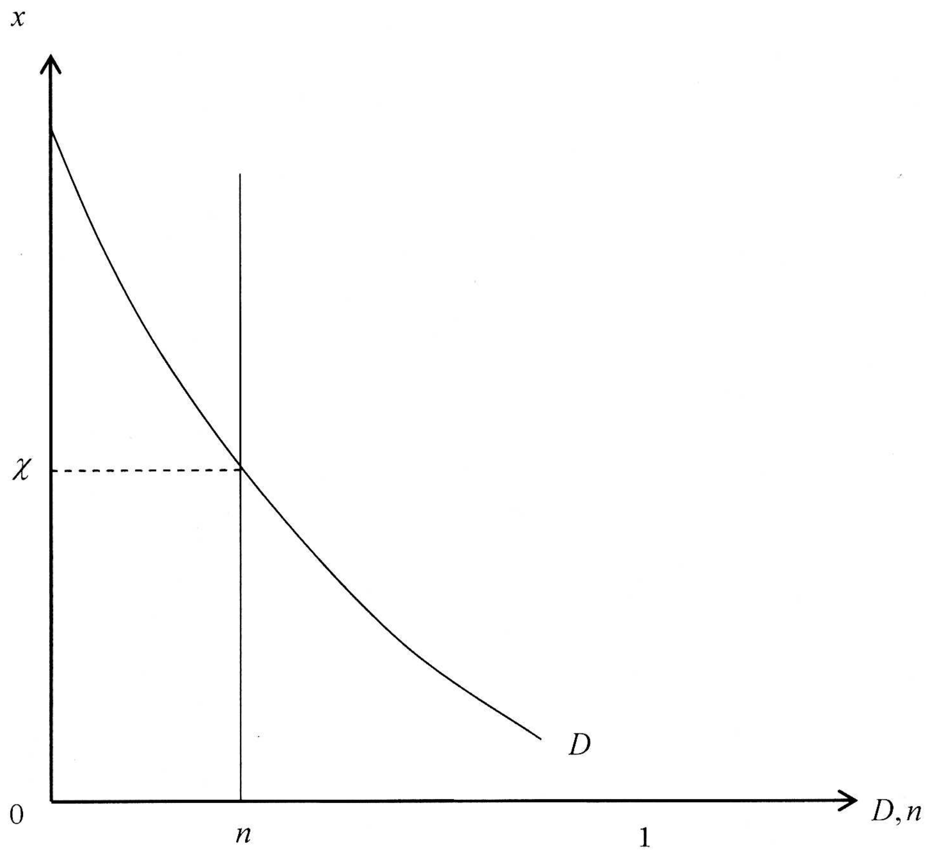


Figure 4.a: The optimal effort level when (14) is binding

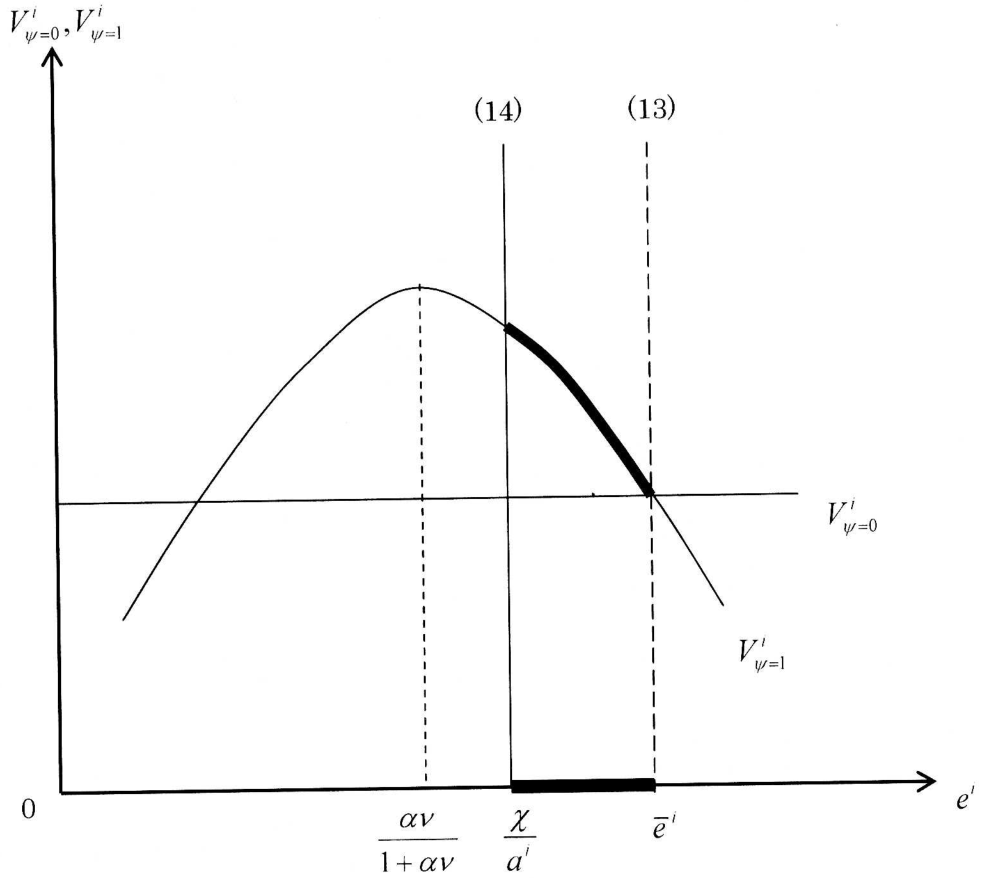


Figure 4.b: The optimal effort level when (14) is not binding

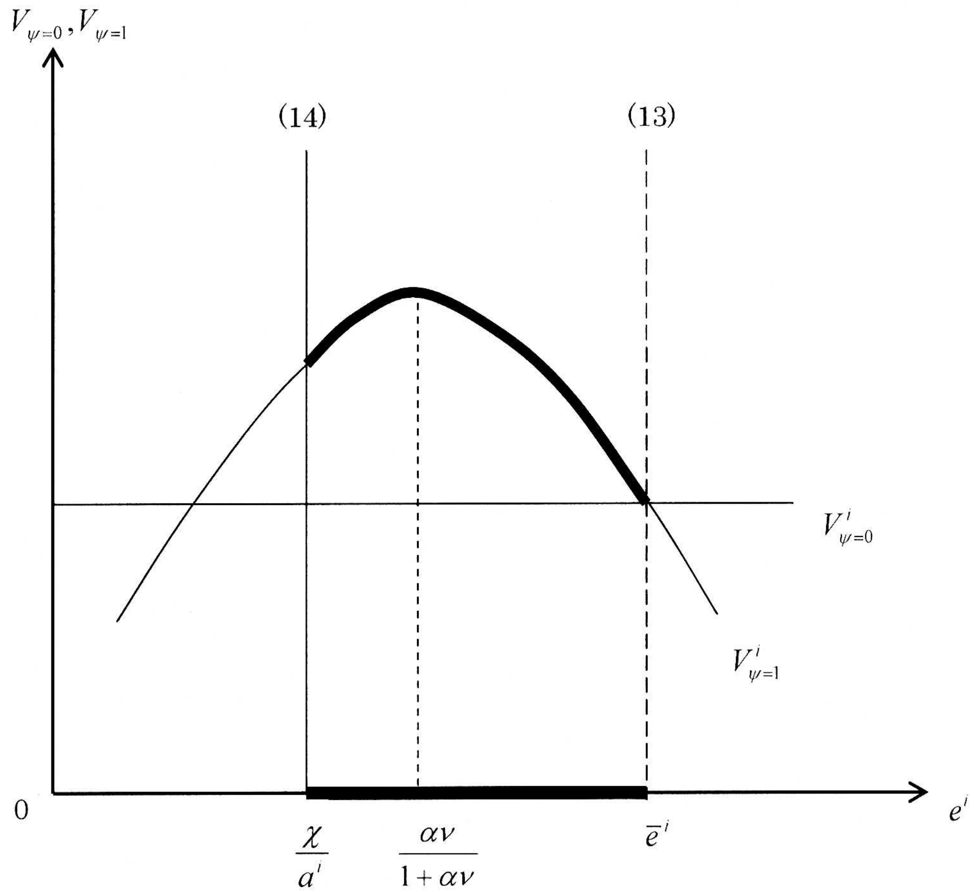


Figure 5.a: Effort levels and learning outcomes in the case of small n and high χ

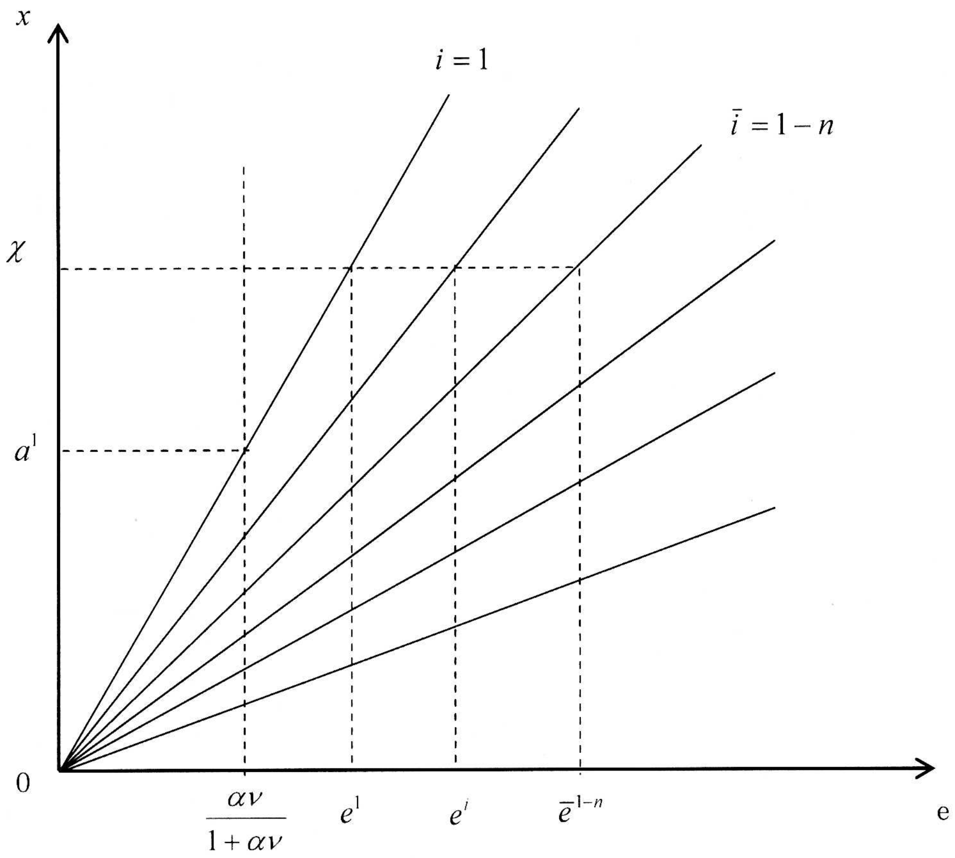


Figure 5.b: Effort levels and learning outcomes in the case of large n and low χ