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Competing Process and Quality Innovation and the Duration of Product Cycles

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Abstract

This paper develops a growth model in which product cycles arise endogenously from investment in process and quality innovations. Incumbent firms invest in productivity improvements with the aim of reducing unit production costs. Market entrants develop new products of superior quality in order to capture the market from vintage product lines. The competing objectives of the two types of innovation generate product cycles within an environment of creative destruction, as new products displace old and are then manufactured using production technologies that are continuously refined. Investigating the relationship between innovation incentives and the average duration of product cycles, we find that there are three stable patterns of product evolution: productivity growth alone, quality growth alone, and product cycles with both types of growth. Stable product cycles are more likely to occur when labor productivity is higher in process innovation than it is in quality innovation and incremental quality improvements are small.

JEL Classifications: 031, 041

Key words: product cycles, product cycle duration, process innovation, quality innovation, product evolution, endogenous growth

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1 Introduction

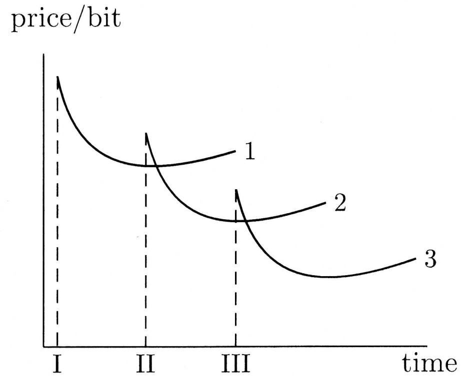
In a modern industrial society, research and development (R&D) is critical both for the survival of firms in a competitive market place and for the growth of the economy. This competition induced R&D activity has many dimensions, including the creation of new markets through the invention of new products, the improvement of product quality, and the refinement of production technology. In the battle for survival, incumbent firms focus on reducing production costs, while new firms entering the market invent products to replace vintage product lines, leading to a process of creative destruction from which product cycles emerge. In this paper, we investigate how competition between advances in production technology and improvements in product quality determines the existence and duration of product cycles.

As a stylized example, consider the product cycles that have coincided with incremental improvements in product quality and falling production costs in the market for portable audio devices. This market was first created with the invention of the Sony Walkman, a portable cassette player, in 1979.[1] Rival firms soon entered the market with similar products manufactured at lower cost, however, leading to fierce price competition. In time, as the magnetic cassette was displaced by the compact disk, Sony developed the Diskman, a portable CD player in 1984. Once again, the entrance of rival firms and price competition led to significant cost reductions. Similar product cycles have subsequently occurred in the market for portable audio devices, with the introduction of the Sony Minidisk player in 1992 and SaeHan Information Systems MPMan digital audio player in 1998. Current product cycles feature the merging of digital audio and video technologies in the iPod line produced by Apple.

The product cycle described above is characteristic of many manufacturing industries. Another example can be taken from the semiconductors industry, where simple comparisons of quality-adjusted prices across density categories are possible, as mem-

[1]See Haine (2009) for a brief history of the Sony Walkman.

Figure 1: Stylized DRAM price patterns



ory chips within a specific class and density category are generally considered to be homogeneous. Grimm (1998) studies the quality-adjusted price patterns of dynamic random access memory (DRAM) chips for four densities, 4kb, 16kb, 64kb, and 256kb, over the period from 1974 to 1994, and finds that price trends exhibit a cyclical pattern. Figure 1 reproduces the stylized representation of these price patterns given in Flamm (1993). The three curves describe quality-adjusted price trends, measured in price per bit, for three memory chip densities, introduced at times I, II, and III. After the introduction of a higher density (quality) chip, price initially falls quickly before flattening out. This pricing pattern may reflect a competition induced decline in the price-cost markup, increases in the scale of production, or improvements in the production process that lead to cost reductions (Grimm 1998).[2]

In this paper, we introduce a framework in which competition between quality and process innovation potentially generates product cycles that produce pricing patterns similar to those illustrated in Figure 1. In particular, we combine elements of the quality ladders (Grossman and Helpman 1991) and in-house process innovation (Smulders and van de Klundert 1995; Peretto 1996) literatures to model quality and process innovation as separate and competing activities. The distinction between these two types of R&D is important. While process innovation is undertaken by incumbent

[2]See Irwin and Klenow (1994) for a discussion of the learning curve in semiconductor production.

firms for the purpose of reducing production costs through the development of new technologies, the creation of production expertise, and the reduction of defective output, quality innovation is associated with development of new product designs that supplant vintage product lines. Endogenous product cycles are potentially generated by the tension arising between the creative destruction associated with new product lines and the efficiency gains coinciding with improvements in production technologies.

We use the framework to investigate patterns of product evolution and find that long-run equilibrium may be characterized by corner solutions with either productivity or quality growth alone, or an interior equilibrium with product cycles that feature both productivity and quality growth. Stable product cycles are more likely to occur when labor productivity is higher in process innovation than it is in quality innovation, and both technology spillovers from market leaders to rival firms and incremental quality improvements are relatively small. We conclude, however, that stable product cycles are never the socially optimal pattern of product evolution.

In equilibrium, product cycle duration is determined stochastically by the interaction between quality and process innovation, generating heterogeneity across industries: some industries experience short, and others long, product cycles. Accordingly, although average product cycle duration is constant and average product price falls at a constant rate in equilibrium, adjustments in the price-cost markup set by industry leaders along the product cycle create within industry pricing patterns that are similar to those illustrated in Figure 1. In particular, technology spillovers from industry leaders to rival producers of vintage product lines diminish over the length of the product cycle, causing the quality-adjusted prices set by industry leaders to fall at decreasing rates.

Our research theme is similar with those of Klepper (1996) and Cohen and Klepper (1996). These papers develop dynamic models of a product life cycle in which the number of firms in an industry expands, peaks, and then declines before stabilizing.

Incumbent firms invest in quality innovations that increase the price consumers are willing to pay for a good and process innovations that decrease the average cost of producing the good. The share of investment in process innovation is shown to increase over the life of the industry, as the returns to process innovation rise, and the benefits to quality innovation fall, with the increasing market shares and production scales of incumbent firms that are able to remain in the market. Our framework differs from these studies in that we are interested in the patterns of product evolution generated by successive product cycles, rather than the dynamics of market structure that occur over a single product cycle. In addition, while Klepper (1996) and Cohen and Klepper (1996) assume that firms only enjoy the benefits of innovation during the period of investment, we focus on how the relationship between incentives to invest in process innovation and the risk of market entry in an environment of creative destruction combine to produce product cycles.

This paper is also closely related to studies that develop models with cyclical behavior between different types of R&D activity. For example, Segerstrom (1991), Davidson and Segerstrom (1998) and Cheng and Tao (1999) consider models in which interaction between the incentives for investment in quality innovation and quality imitation causes industries to cycle between monopoly and a colluding duopoly. In addition, Cheng and Dinopoulus (1996) consider the duration of product cycles by making a distinction between breakthrough quality innovations and quality improvements, and find that product cycle duration is negatively related with trend growth. Our paper differs from these studies, however, in that we consider how the relationship between incentives for investment in quality and process innovation may lead to endogenous product cycles.

The remainder of this paper is organized as follows. In Section 2, we introduce our basic model of endogenous growth. Section 3 investigates the characteristics of long-run equilibria, and Section 4 considers socially optimal patterns of product evolution.

Section 5 provides some brief concluding remarks.

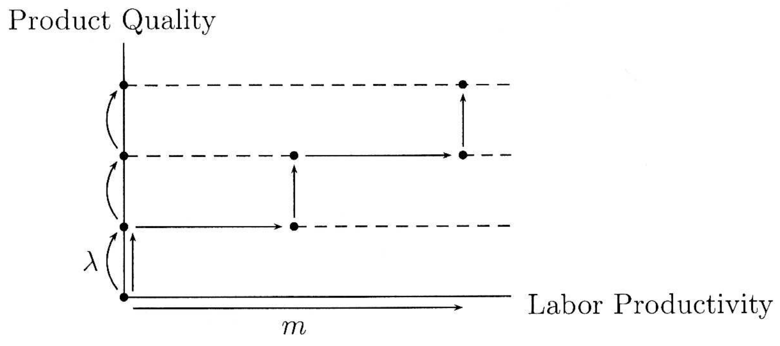
2 The model

Consider an economy with three economic activities: production (X), process innovation (M), and quality innovation (Q). The production sector consists of a unitary mass of industries, indexed by ω, within each of which firms produce goods for consumption and compete according to Bertrand competition. Process innovation refers to the invention of production technologies that improve the productivity of incumbent firms thereby reducing production costs. Quality innovation, on the other hand, refers to the market entry of rival firms with new product designs that improve on the quality of existing product lines. The sole factor of production is labor supplied inelastically by households.

An illustration of the product space for an industry is provided in Figure 2, where the vertical axis measures the level of the product quality, and the horizontal axis measures labor productivity in production. We define the state-of-the-art of a given industry as the product that has the highest available quality and that is produced using the lowest cost production technology, and refer to the firm producing the state-of-the-art as the industry leader. Focusing on discrete quality and continuous productivity improvements, a successful research effort in the quality innovation sector leads to the introduction of a new product line with a quality of λ times the quality level of the current state-of-the-art. The dashed lines in Figure 2 denote the product lines associated with different quality levels, and the parameter $\lambda > 1$ is the vertical distance between them (Grossman and Helpman 1991). In contrast, process innovations for incumbent firms result in an improvement in the effective productivity of workers, $m > 0$, producing the current state-of-the-art, as illustrated by the horizontal arrows (Smulders and van de Klundert 1995; Peretto 1996).

The product space described in Figure 2 suggests three possible patterns for prod-

Figure 2: The process and quality dimensions of product evolution



uct evolution. In the first, consecutive quality innovations improve the quality of the state-of-the-art, but no process innovation occurs, as described by the arced arrows running up the the vertical axis. In the second, there are consecutive process innovations, but quality innovation never occurs, as shown by the arrow running along the horizontal axis. The third potential pattern of product evolution consists of continuous process innovation with the intermittent introduction of quality improvements. In the following sections, we develop a framework that enables an investigation of the conditions required for each of these patterns to arise in a stable long-run equilibrium.

2.1 Households

The demand side of the economy consists of a representative dynastic household that chooses an optimal expenditure path with the objective of maximizing lifetime utility. Intertemporal preferences are described by

$$U = \int_0^\infty e^{-\rho t} \log u(t) dt, \tag{1}$$

where $\rho \in (0, 1)$ is the subjective discount rate, and instantaneous utility $u(t)$ takes the form of a quality-augmented Dixit-Stiglitz consumption index with a unitary elasticity of substitution between industries:

$$\log u(t) = \int_0^1 \log \left(\sum_{j(\omega)} \lambda^{j(\omega)} x(m, \omega, t) \right) d\omega. \tag{2}$$

Product quality and quantity are denoted by $\lambda^{j(\omega)}$ and $x(j, \omega, t)$. Instantaneous utility is increasing in $j(\omega)$, the number of quality innovations that have been introduced by time t in industry ω, and consumers therefore prefer higher quality products.

Intertemporal optimization requires that the representative household select an expenditure path that maximizes lifetime utility (1) subject to the following intertemporal budget constraint: $B(0) \geq \int_0^\infty e^{-R(t)} E(t) dt$, where $R(t) = \int_0^t r(s) ds$ is the cumulative interest factor, $r(t)$ is the rate of return, $E(t)$ is household expenditure, and $B(0)$ is the present value of the future flow of household income plus the initial value of household assets. It is well known that the solution to this problem is the optimal expenditure-saving path described by the following Euler equation (Grossman and Helpman, 1991):

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \tag{3}$$

where a dot over a variable denotes time differentiation. Henceforth, we set expenditure as the model numeraire, and $r = \rho$ at all moments in time. For the remainder of the paper we suppress time notation when doing so does not cause confusion.

With a unitary elasticity of substitution across industries, households allocate expenditures evenly across all product lines, and the demand function for the state-of-the-art in industry ω becomes

$$x(j, \omega, t) = \frac{1}{p_x(j, \omega, t)}, \tag{4}$$

where $p_x(j, \omega, t)$ is price. In each industry, households consume only the good with the lowest quality-adjusted price, and demand is therefore zero for all products that are not state-of-the-art.

2.2 Production

Firms in the production sector employ labor with an industry-specific technology. In particular, the production function is

$$x(\omega) = m(\omega)L_X(\omega), \tag{5}$$

where $m(\omega)$ is current productivity and $L_X(\omega)$ is firm-level employment in production.

Firms operating in the same industry compete according to Bertrand competition and, as a result, the profit maximizing price of the industry leader, producing the state-of-the-art, is a limit price that is set just low enough to force the closest rival firm out of the market. Denoting the productivity of the closest rival firm as $\overline{m}(\omega)$, we find that the industry leader sets a quality-adjusted price that is just equal to the marginal cost of the closest rival firm, $p_x(\omega) = w\lambda/\overline{m}(\omega)$, where w is the economy-wide wage rate. This limit price allows the industry leader to earn operating profit on sales equal to $\pi(\omega) = p(\omega)x(\omega) - wL_X(\omega)$ each period:

$$\pi(\omega) = 1 - \frac{\overline{m}(\omega)}{\lambda m(\omega)}, \tag{6}$$

where we have used the demand function (4), the limit pricing rule, $p_x(\omega) = w\lambda/\overline{m}(\omega)$, and the marginal cost of the industry leader, $w/m(\omega)$.

2.3 Process innovation

Following Peretto and Connolly (2007), productivity growth may arise within a given industry as the result of new production technologies that are invented by independent R&D firms in the process innovation sector and then sold to industry leaders in the production sector. At each moment in time, the market leader in a given industry determines its optimal purchases of productivity improvements with the objective of maximizing firm value. The total cost of these purchases is $p_m(\omega)\dot{m}(\omega)$, where $p_m(\omega)$ is the unit price and $\dot{m}(\omega)$ is the mass of productivity improvements purchased. Accordingly, in view of operating profit on sales (6), in an industry with positive productivity growth, the market leader earns instantaneous profits equal to

$$\Pi(\omega) = 1 - \frac{\overline{m}(\omega)}{\lambda m(\omega)} - p_m(\omega)\dot{m}(\omega). \tag{7}$$

The incentive to invest in new process innovations is clear. An improvement in the productivity $m(\omega)$ of the incumbent firm decreases the limit price to marginal cost ratio, $\overline{m}(\omega)/\lambda m(\omega)$, raising operating profit on sales (6), and increasing firm value.

While advances in production technology are known to all firms in the industry, rival firms have no incentive to purchase new process innovations for the production lines of vintage products, as they would still have to set a higher quality-adjusted price than that of the state-of-the-art, and would therefore not be able to capture a positive share of the market. There is a feedback effect, however, from the industry leader to the closest rival firm in that some of the productivity improvements made on the state-of-the-art production line may be immediately adaptable to vintage products. In particular, we assume that technology spillovers generate a productivity level for the nearest rival firm of

$$\overline{m}(\omega) = m(\omega)^{\gamma}, \tag{8}$$

where $\gamma \in (0, 1)$ captures the diminishing feasibility with which new production technologies can be adapted to the production lines of vintage products.

Competitive process innovation firms develop productivity improvements for sale to firms in industry ω using the following R&D technology:

$$\dot{m}(\omega) = \alpha m(\omega) L_M(\omega), \tag{9}$$

where $\alpha > 0$ is a productivity parameter, $m(\omega)$ is the current stock of industry-specific technical knowledge, and $L_M(\omega)$ is total labor employed in the development of new production processes for industry ω. With this R&D technology, free entry into the process innovation sector and a continuous mass of production industries leads to a unit price for productivity improvements that matches the cost of development: $p_m(\omega) = w/(\alpha m(\omega))$.

Firm value for the industry leader is equal to the present value of expected profit flows:

$$v(\omega) = \int_0^\infty e^{-\int_0^t r(s)+\iota(\omega,s)ds}\Pi(\omega, t)dt, \tag{10}$$

where $\iota(\omega)$ is the industry-specific risk associated with the potential loss of the market to a rival firm entering with a newly developed state-of-the-art product design. Assuming that the industry leader accounts for technology spillovers to the nearest rival firm (8), the instantaneous demand for productivity improvements in a given industry is described by the following asset-pricing condition:

$$\rho + \iota(\omega) \geq \frac{1}{p_m}\frac{\partial \Pi(\omega)}{\partial m} + \frac{\dot{p_m}(\omega)}{p_m(\omega)}, \tag{11}$$

where we have used $r = \rho$. When the industry leader exhibits positive productivity growth, the return on investment in productivity improvements must equal the risk-free interest rate (ρ) plus an adjustment for the risk associated with market entry $\iota(\omega)$. Otherwise, $L_M(\omega)$ is set equal to zero, and the industry exhibits no productivity growth.[3]

2.4 Quality innovation

We next turn to the quality innovation sector where competitive firms invest in R&D with the aim of entering the market with new product designs that improve upon the qualities of current state-of-the-art products. Each new product design includes a quality improvement and a production process that adopts all of the quality improvements and process innovations that have been introduced to date in the respective industry. A new product design therefore has a quality that is one increment greater than the current state-of-the-art and reproduces the productivity level $m(\omega)$ of the current industry leader.

A new quality innovation is successfully developed in industry ω with probability $\iota(\omega)dt$ if research is undertaken for a time interval of dt at an intensity of $\iota(\omega)$. This research intensity requires the employment of $\beta L_Q(\omega)$ units of labor, where $\beta > 0$ is a productivity parameter. With free entry and exit, there is active quality innovation when the expected cost of successfully developing a new quality innovation is equal to the present value of the potential profit stream that is earned with successful market entry (10): $v(\omega) = w/\beta$. Taking the time derivative of this free entry conditions yields an asset-pricing condition for investment in quality innovation:

$$\rho + \iota(\omega) \geq \frac{\Pi(\omega)}{v(\omega)} + \frac{\dot{v}(\omega)}{v(\omega)}, \tag{12}$$

where we have used $r = \rho$, and $\iota(\omega)$ is once again the risk that a subsequently developed quality improvement allows a later entrant to capture the market. The rate of return on quality innovation must equal the sum of the risk-free interest rate (ρ) and the risk premium $\iota(\omega)$ if the industry displays a positive rate of quality growth.

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[3]Given $r = \rho$, the transversality condition associated with optimal purchases of productivity improvements is $\lim_{t\to\infty} e^{-\int_0^t \rho+\iota(\omega,s)ds} p_m(\omega,t)m(\omega,t) = 0$.

3 Long-run production evolution

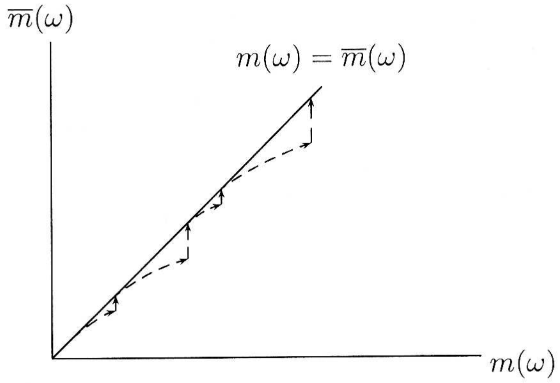
We now close the model and characterize the patterns of product evolution associated with various long-run equilibria. As discussed above, we are interested in three possible patterns of production evolution: process innovation alone, quality innovation alone, and product cycles with both process and quality. This section investigates and compares the conditions required for each pattern of product evolution to arise.

3.1 Competing process and quality innovation

In order to facilitate a simple interpretation of product evolution, we begin by introducing a variable to describe the length of product cycles within each industry. We then combine the labor demands from each industry in order to derive a labor market condition that determines the equilibrium wage rate. Finally, we investigate the local dynamics associated with each of the three patterns of product evolution.

Figure 3 illustrates the evolution of production technology for an industry with both active process and quality innovation. The horizontal and vertical axes respectively measure the productivity of the industry leader and the productivity of the closest rival firm. When an entering firm captures the market with a new quality innovation, its initial production technology is the same as that of its nearest rival. Thus, each new entrant begins with a productivity level on the $m(\omega) = \overline{m}(\omega)$ locus. As the new incumbent, the firm then invests in process innovations that raise both its own productivity and that of the nearest rival firm, although at a diminishing rate, since $\overline{m}(\omega) = m(\omega)^{\gamma}$. This deterministic improvement in production technology is described by a rightward movement along a dashed curve. The level of productivity growth associated with this incumbent firm depends on the length of time before the next market entry, the timing of which is determined stochastically. During this time interval there may be no productivity growth if subsequent entry is immediate, and infinite productivity growth if subsequent entry never occurs. When market entry

Figure 3: Product Cycle Duration



does occur, the industry jumps vertically back to the $\overline{m}(\omega) = m(\omega)$ locus.

To capture the dynamics of product quality and productivity described above, we introduce a new variable

$$\chi(\omega) \equiv \overline{m}(\omega)/m(\omega), \tag{13}$$

which must, by definition, take values between zero and one. We refer to (13) as *product cycle duration* as, at in given moment in time, the ratio of rival-firm productivity to market-leader productivity provides an indication of the length of the current product cycle. If χ is close to one, the current product cycle is comparatively new. If χ is close to zero, the current product cycle is comparatively old. The dynamics of χ provide a means of investigating product evolution. At each moment in time, industry-leader productivity changes according to (9). Similarly, the expected change in rival-firm productivity is $\dot{\overline{m}}(\omega) = (m(\omega) - \overline{m}(\omega))\, \iota(\omega) + \gamma m^{\gamma-1}\dot{m}$, where the first term shows that the current industry leader becomes the closest rival firm if a firm enters the market with a new quality innovation and the second term captures technology spillovers from the market leader to the closet rival firm. Substituting these differential equations into the time derivative of $\chi(\omega) \equiv \overline{m}(\omega)/m(\omega)$, we find that the evolution of product cycle duration in industry ω is governed by

$$\dot{\chi}(\omega) = (1 - \chi(\omega))\iota(\omega) - \chi(\omega)(1 - \gamma)\frac{\dot{m}(\omega)}{m(\omega)}. \tag{14}$$

Thus, product cycle duration is determined by stochastic and deterministic processes.

3.2 Labor Market

Next we derive parameter conditions for patterns of product evolution that are consistent with full employment in the labor market. Assuming, for the moment, that there is both active process and quality innovation in the economy, the labor market clears for

$$L = L_X + L_M + L_Q, \tag{15}$$

where $L_X \equiv \int_0^1 L_X(\omega)d\omega$, $L_M \equiv \int_0^1 L_M(\omega)d\omega$, and $L_Q \equiv \int_0^1 L_Q(\omega)d\omega$ are the average labor demands from production, process innovation, and quality innovation.

Substituting the limit price $p_x(\omega) = w\lambda/\overline{m}(\omega)$ into demand (4) and setting the result equal to supply (5), the average demand for labor from production is

$$L_X = \frac{\chi}{w\lambda}, \tag{16}$$

where $\chi \equiv \int_0^1 \chi(\omega)d\omega = \int_0^1 \overline{m}(\omega)/m(\omega)d\omega = \overline{m}/m$ is the average duration of product cycles across the economy. Note that m and $\overline{m}$ are not average values, but rather are the industry leader and closest rival firm productivity levels associated with the average employment levels L_X, L_M, and L_Q. Thus, $m \neq \int_0^1 m(\omega)d\omega$ and $\overline{m} \neq \int_0^1 \overline{m}(\omega)d\omega$.

While labor employment in production is determined independently of the dynamics of product evolution, employment in innovation is closely related to the pattern of product development. In an equilibrium with product cycles, the asset-pricing conditions (11) and (12) both must bind. Accordingly, two conditions for the relationship between average industry employment levels in process and quality innovation are

$$\rho + \beta L_Q = R_M \equiv \frac{\alpha(1-\gamma)\chi}{\lambda w} - \alpha L_M + \frac{\dot{w}}{w}, \tag{17}$$

$$\rho + \beta L_Q = R_Q \equiv \frac{\beta}{w} - \frac{\beta\chi}{\lambda w} - \beta L_M + \frac{\dot{w}}{w}, \tag{18}$$

where we have used $\overline{m} = m^{\gamma}$, $p_m = w/(\alpha m)$, $v = w/\beta$, $\iota = \beta L_Q$, $\dot{m} = m\alpha L_M$, and $\chi \equiv \overline{m}/m$ with (7) in (11) and (12). The lefthand sides of these asset pricing conditions are the risk-adjusted market rates of return, while the righthand sides are the rates of return to process innovation (R_M) and to quality innovation (R_Q).

3.3 A Pattern of Process Innovation

We begin by considering the stability of a long-run equilibrium with process innovation alone. In this case, there is no investment in new product designs, and the asset-pricing condition for investment in quality improvements (18) does not bind, indicating that $L_Q = 0$. As we are interested in long-run equilibria that feature a constant allocation of labor across sectors, we consider steady states characterized by $\dot{L_X} = \dot{L_M} = 0$ and $\dot{\chi}/\chi = \dot{w}/w$, from (16).

We begin by deriving differential equations to describe the evolution of the wage rate and the motion of average product cycle duration. Substituting (15) and (16) into (17), and using $\iota = \beta L_Q = 0$ and $\dot{m} = \alpha m L_M$ with (15) in (14) yields the following differential equations for w and χ:

$$\frac{\dot{w}}{w} = \rho - \alpha(1-\gamma)L + \alpha(2-\gamma)L_M, \qquad \frac{\dot{\chi}}{\chi} = -\alpha(1-\gamma)L_M. \tag{19}$$

The stability of steady-state equilibrium can be discerned through an investigation of the local dynamics of employment in process innovation around $\dot{L_M} = 0$:

$$\dot{L_M} = (L - L_M)\left(\frac{\dot{w}}{w} - \frac{\dot{\chi}}{\chi}\right) = (L - L_M)\left(\alpha(1 + 2(1-\gamma))L_M - \alpha(1-\gamma)L + \rho\right), \quad (20)$$

where we have used the time derivative of (16) with $L_M = L - L_X$. Given that L_M is a control variable, as $d\dot{L}_M/dL_M = \alpha(1+2(1-\gamma))(L-L_M) > 0$ around $\dot{L}_M = 0$, we find that the economy jumps immediately to a dynamic path satisfying $\dot{\chi}/\chi = \dot{w}/w$, when firms are not investing in quality improvements. Then, the wage rate and average product cycle duration converge towards zero together.

Setting (20) equal to zero produces the steady-state level of employment in process innovation, and thus the equilibrium rate of process innovation is

$$\frac{\dot{m}}{m} = \alpha L_M = \frac{\alpha(1-\gamma)L - \rho}{1 + 2(1-\gamma)}. \quad (21)$$

The basic features of this innovation rate are standard, with an increase in the labor endowment or the productivity of labor in process innovation raising the innovation rate, and an increase in the interest rate or the feasibility of adapting new production technologies to vintage product lines lowering the innovation rate.

3.4 A Pattern of Quality Innovation

In this section we next investigate the stability of long-run equilibria for which only quality innovation occurs. In this case, as firms are not investing in productivity improvements, the asset-pricing condition for investment in process innovation (17) does not bind, and $L_M = 0$.

Starting with the dynamics of the wage rate, we find that when there is active quality innovation, regardless of whether process innovation occurs or not, substituting

(15) and (16) into (18) yields the following wage dynamics:

$$\dot{w} = w(\beta L + \rho) - \beta. \tag{22}$$

As a result, with active quality innovation, the wage rate is determined independently of average product cycle duration (χ). Since the wage rage is a control variable, and $\partial\dot{w}/\partial w = \beta L + \rho > 0$, it jumps immediately to its steady-state value of $w = \beta/(\beta L + \rho)$. Then, setting $L_M = 0$ in (14) and reorganizing the result, the dynamics of average product cycle duration can be described by the following differential equation:

$$\dot{\chi} = (1 - \chi)\beta L_Q = (1 - \chi)\beta \left(L - \frac{\chi}{\lambda w} \right), \tag{23}$$

where we have used (15) and (16). Evaluating the derivative of this differential equation around $\chi = 1$ yields $d\dot{\chi}/d\chi = -\beta(\lambda w L - 1)/(\lambda w) < 0$. We conclude, therefore, that an equilibrium with quality innovation alone is also stable.

Setting L_M and $\dot{w}$ equal to zero in (18) and using $w = \beta/(\beta L + \rho)$, we find that the equilibrium rate of quality innovation is

$$\iota = \frac{(\lambda - 1)\beta L - \rho}{\lambda}. \tag{24}$$

In this case, the rate of innovation is increasing in the labor endowment, labor productivity in quality innovation, and the size of the quality increment, but decreasing in the interest rate, as found in Grossman and Helpman (1991).

3.5 A Pattern of Product Cycles

Now that we have confirmed the stability of long-run equilibria that feature either productivity growth or quality growth alone, we consider the conditions required for a long-run pattern of product evolution with product cycles.

Substituting (15) and (16) into (18), we first confirm that wage dynamics once again follow (22), with the wage rate jumping immediately to $w = \beta/(\beta L + \rho)$, allowing us to focus on the dynamics of average product cycle duration. In addition, in Sections 3.4 and 3.5 we found that positive rates of productivity and quality growth respectively require $\alpha(1-\gamma)L > \rho$ and $(\lambda-1)\beta L > \rho$. As we are interested in long-run equilibria with positive growth, we assume that these resource constraints are satisfied for the remainder of the paper.

Product cycles require simultaneous investment in both types of innovation, implying that the asset-pricing conditions (17) and (18) must both bind, and that the rates of return to process innovation and quality innovation must equalize. Setting $R_M = R_Q$ and rearranging the result, we obtain the average level of employment in process innovation as

$$L_M^* = \frac{\beta(\chi - \chi_M)}{(\alpha - \beta)\chi_M w}, \tag{25}$$

where the threshold value $\chi_M \equiv \lambda\beta/(\alpha(1-\gamma)+\beta) > 0$ is a boundary on the range of average product cycle duration over which there is active investment in process innovation. This condition shows the combinations of L_M and χ for which the rates of return to process and quality innovation equalize and the labor market clears.

A similar employment condition can be obtained for quality innovation by substituting (25) into (18):

$$L_Q^* = \frac{\alpha(2-\gamma)(\chi_Q - \chi)}{(\alpha - \beta)\lambda w}, \tag{26}$$

where the threshold value $\chi_Q \equiv \beta\lambda(\alpha L + \rho)/(\alpha(2-\gamma)(\beta L + \rho)) > 0$ is a boundary on the range of average product cycle duration over which there is active investment in quality innovation. This employment condition shows the combinations of L_Q and χ that ensure equal rates of return to process and quality innovation.

The ranking of the thresholds for average product cycle duration, and thus the feasibility of active employment in both types of innovation, and equal rates of return, depends on the relative productivities of labor in process and quality innovation:

Lemma 1 *Active investment in both process and quality innovation is possible in an interior equilibrium when (i) $\chi \in (\chi_Q, \chi_M)$ for $\alpha < \beta$, and (ii) when $\chi \in (\chi_M, \chi_Q)$ for $\alpha > \beta$. The interior equilibrium is not defined for $\alpha = \beta$.*

Proof: Sign L_M^* and L_Q^* using $\chi_Q - \chi_M = (\alpha - \beta)(\alpha(1-\gamma)L - \rho)w\chi_M/((\alpha\beta(2-\gamma))$, with $\alpha(1-\gamma)L > \rho$, as required for positive productivity growth.

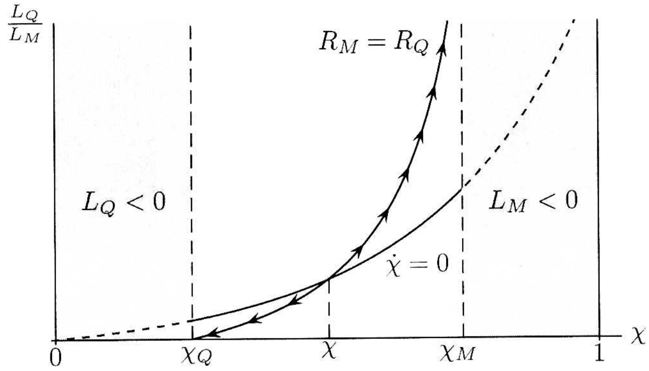
Combining (25) and (26), we calculate the equilibrium ratio of employment in quality and process innovation as follows:

$$\frac{L_Q}{L_M} = \frac{\alpha(2-\gamma)(\chi_Q - \chi)}{(\alpha(1-\gamma)+\beta)(\chi - \chi_M)}. \tag{27}$$

When this labor market clearing condition is satisfied, rates of return equalize, and there are positive employment levels for both types of innovation. The labor market clearing condition is illustrated by the $R_M = R_Q$ curve in Figures 4 and 5, with a positive slope for $\alpha/\beta < 1$ and a negative slope for $\alpha/\beta > 1$, matching with cases (i) and (ii) in Lemma 1. The shaded areas of each figure indicate the one type of innovation is not feasible: in Figure 4 there is no productivity growth if $\chi > \chi_M$ and no quality growth if $\chi < \chi_Q$; and in Figure 5 there is no productivity growth if $\chi < \chi_M$ and no quality growth if $\chi > \chi_Q$. Above the $R_M = R_Q$ curve, the rate of return to process innovation is greater than the rate of return to quality innovation, and the employment ratio falls. Conversely, below the $R_M = R_Q$ curve, the rate of return to quality innovation is greater and L_Q/L_M rises. These investment dynamics ensure that the economy always lies on the $R_M = R_Q$ curve when there are positive rates of productivity and quality growth.

Next, we consider the stability of interior equilibria with product cycles through an

Figure 4: $\alpha/\beta < 1$



investigation of the dynamics of average product cycle duration. Substituting average employment levels for quality and process innovation, $\iota = \beta L_Q$ and $\dot{m}/m = \alpha L_M$, into (14), the evolution of average product cycle duration is found as

$$\dot{\chi} = L_M \left((1-\chi)\beta \frac{L_Q}{L_M} - \chi\alpha(1-\gamma) \right). \tag{28}$$

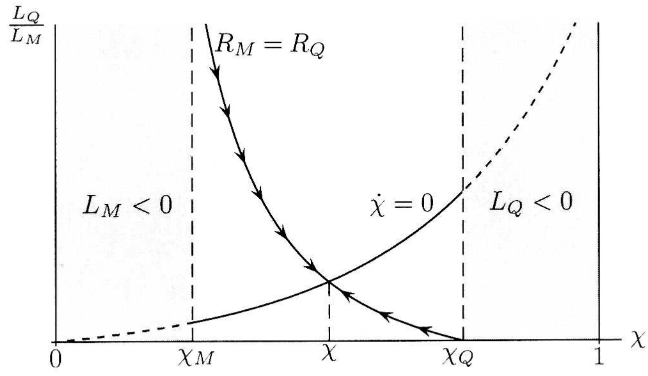
Naturally, the motion of average product cycle duration depends on relative investment levels. Setting $\dot{\chi} = 0$, and once again using (15), we obtain a steady-state condition for the innovation employment ratio and average product cycle duration:

$$\frac{L_Q}{L_M} = \frac{\alpha(1-\gamma)\chi}{\beta(1-\chi)}. \tag{29}$$

The steady-state locus has a positive slope, as shown by the $\dot{\chi} = 0$ curve in Figures 4 and 5. Evaluating the partial derivative of (28) with respect to χ, around $\dot{\chi} = 0$, yields $\partial\dot{\chi}/\partial\chi = -(\beta L_Q + \alpha(1-\gamma)L_M) < 0$, and average product cycle duration is therefore rising for values of χ to the left of the $\dot{\chi} = 0$ curve, and falling for values of χ to the right of the $\dot{\chi} = 0$ curve.

A closer look at (27) and (29) reveals that the threshold level of average product cycle duration for investment in process innovation (χ_M) is the key determinant of the existence of an interior equilibrium, since the $\dot{\chi} = 0$ curve lies strictly above

Figure 5: $\alpha/\beta > 1$



the $R_M = R_Q$ curve at $\chi = \chi_Q$ in both Figures 4 and 5. In particular, the $\dot{\chi} = 0$ and $R_M = R_Q$ curves tend to infinity as they approach the vertical asymptotes at $\chi = 1$ and $\chi = \chi_M$. As such, $\chi_M < 1$ is a necessary condition for the existence of an interior equilibrium with positive rates of productivity and quality growth. When this condition is not satisfied, the economy converges to a corner solution with productivity growth if $\alpha/\beta < 1$, and to a corner solution with quality growth if $\alpha/\beta > 1$.

Combining information on the threshold values for average product cycle duration, χ_M and χ_Q, with the results of Lemma 1, we obtain the following proposition:

Proposition 1 *The economy converges to a long-run equilibrium with stable product cycles for* $\alpha/\beta > 1$ *and* $\chi_M < 1$.

The stability of an interior equilibrium with product cycles depends on how changes in average product cycle duration affect the relative rates of return to investment in innovation. First, consider the unstable case illustrated in Figure 4 for $\alpha/\beta < 1$ and $\chi_M < 1$. Although a unique equilibrium exists, any small shock will lead to unstable product cycles that drive the economy to a corner solution with either productivity growth at $\chi = 0$ or quality growth at $\chi = 1$, as described in Sections 3.3 and 3.4. These dynamics derive from the positive and negative relationships that arise between investment in quality and process innovation and average product cycle duration for $\alpha/\beta < 1$. Specifically, if product cycle duration is initially below its steady-state level, a fall in χ lowers employment in quality innovation, and raises employment in process innovation leading to further decreases in χ, until the economy converges to a corner solution with process innovation alone. Alternatively, if product cycle duration is initially above its steady-state level, employment dynamics work in the opposite direction, with employment rising in quality innovation and falling in process innovation, until only quality innovation remains.

On the other hand, in the stable case depicted in Figure 5 for $\alpha/\beta > 1$ and $\chi_M < 1$, there are positive and negative relationships between investment in process and quality innovation and average product cycle duration, and investment dynamics accordingly push the economy towards an interior equilibrium with product cycles. For example, an increase in χ raises employment in process innovation, but lowers employment in quality innovation, causing a subsequent decrease in χ. In conclusion, we find that the long-run pattern of product evolution is most likely to feature stable product cycles when labor productivity is higher in process innovation than it is in quality innovation ($\alpha/\beta > 1$), and when technology spillovers from the market leader to the nearest rival firm (γ) and the discrete quality increment (λ) are relatively small, ensuring that the interior equilibrium exists ($\chi_M < 1$).

We briefly consider the effects of parameter changes on the average duration of product cycles, and obtain the following proposition:

Proposition 2 *With stable product cycles, parameter changes have the following effects on average product cycle duration:* $d\chi/d\alpha < 0$, $d\chi/d\beta > 0$, $d\chi/d\gamma > 0$, *and* $d\chi/d\lambda > 0$.

Proof: See Appendix A.

These results follow basic economic tuition. Parameter changes that raise the relative return to investment in one type of innovation increase employment in that type of innovation with natural results for product cycle duration. A rise in α, or a fall in γ, increases the relative return to process innovation causing χ to fall. A rise in either

β or λ increases the relative return to quality innovation causing χ to rise.

3.6 Social Optimum

This section considers whether stable product cycles are socially optimal. Steady-state instantaneous utility is $\log \mu(t) = I(t) \log \lambda + \log x$, where $I(t) = \int_0^t \iota(s)ds$ is the expected number of quality improvements that have been introduced before time t (Grossman and Helpman 1991). Using the production function (5) and $I(t) = \beta L_Q t$, we have $\log \mu(t) = \beta L_Q t \log \lambda + \log m + \log L_X$, where the first, second, and third terms are the utility derived from the level of product quality, from the stock of technology improvements, and the quantity of goods consumed. With constant employment levels, the time derivative of instantaneous utility yields the rate of economic growth:

$$g \equiv \frac{\dot{\mu}(t)}{\mu(t)} = \beta L_Q \log \lambda + \alpha L_M. \tag{30}$$

The social planner's objective is to maximize $U(t) = \int_t^\infty e^{-\rho\tau}((\log \lambda) I(\tau) + \log m(\tau) + \log L_X(\tau))d\tau$, subject to the technology constraints $\dot{m}(\tau) = m(\tau)\alpha L_M(\tau)$ and $\dot{I}(\tau) = \beta L_Q(\tau)$. We solve the utility maximization problem using the following current value Hamiltonian function: $H = (\log \lambda)I + \log m + \log L_X + \zeta_1 \alpha m L_M + \zeta_2 \beta L_Q$, where ζ_1 and ζ_2 are the shadow values of average productivity and quality improvements. The solution to this optimisation problem reduces to two conditions for the socially-optimal pattern of product evolution:

$$L_X = \frac{\rho}{\alpha}, \qquad\qquad L_X = \frac{\rho}{\beta(\log \lambda)}. \tag{31}$$

The first condition binds for positive productivity growth and the second binds for positive quality growth.

The resource constraints (31) indicate that the socially optimal pattern of product evolution only features productivity and quality growth together in a knife-edge equilibrium with $\alpha/\beta = \log\lambda$. It follows, therefore, that the socially optimal pattern of product evolution will usually be characterized by a corner solution. For example, if $\alpha/\beta > \log\lambda$, a comparison of the resource constraints (31) shows that an equilibrium with process innovation alone allows for a higher level of consumption and a higher rate of growth. On the other hand, when $\alpha/\beta < \log\lambda$, an equilibrium with quality innovation alone is socially optimal. Importantly, rewriting $\chi_M < 1$ from Proposition 1, we find that $\alpha/\beta < (\lambda - 1)/(1 - \gamma)$ is required for stable product cycles to occur. This implies, however, that stable product cycles are never socially optimal, as $\log\lambda$ is strictly smaller than $(\lambda - 1)/(1 - \gamma)$.[4]

Proposition 3 *Product cycles with productivity and quality growth are never optimal.*

4 Conclusion

The survival of firms in a competitive market place is intrinsically tied to innovation activity across several dimensions. In this paper, we have investigated the determinants of product cycles arising within an environment of creative destruction, where firms entering a market develop new products to displace vintage product lines and incumbent firms invest in new technologies that reduce production costs. In particular, we study the relationship between competing incentives for process and quality innovation and the average duration of product cycles across the economy, and establish that there are three potential patterns for product evolution within a given industry: productivity growth alone, quality growth alone, and product cycles with both productivity and quality growth. Parameter conditions determine which pattern arises in long-run equilibrium, with stable product cycles more likely to occur when labor productivity is greater in process innovation than in quality innovation, and both technology spillovers from market leaders to rival firms and incremental quality

[4] As $\log\lambda - (\lambda - 1)/(1 - \gamma)$ is decreasing in λ and negative for $\lambda = 1$, $\log\lambda < (\lambda - 1)/(1 - \gamma)$).

improvements are relatively small. We find, however, that these stable product cycles are never the socially optimal pattern of product evolution.

Appendix A

Equating (27) and (29) yields $(2-\gamma)(1-\chi)(\chi_Q-\chi)\chi_M = (1-\gamma)\lambda\chi(\chi-\chi_M)$. We use this condition to obtain the following steady-state comparative statics:

$$\frac{d\chi}{d\alpha} = -\frac{1}{\alpha H}\left(1+\frac{\beta(\lambda-\chi)}{(\alpha(1-\gamma)+\beta)(\chi-\chi_M)}+\frac{\rho\chi_Q}{(\alpha L+\rho)(\chi_Q-\chi)}\right) < 0,$$
$$\frac{d\chi}{d\beta} = \frac{1}{\beta H}\left(1+\frac{\beta(\lambda-\chi)}{(\alpha(1-\gamma)+\beta)(\chi-\chi_M)}+\frac{\rho\chi_Q}{(\beta L+\rho)(\chi_Q-\chi)}\right) > 0,$$
$$\frac{d\chi}{d\gamma} = \frac{1}{(2-\gamma)H}\left(\frac{2-\gamma}{1-\gamma}+\frac{\alpha\chi+\beta(\lambda-\chi)}{(\alpha(1-\gamma)+\beta)(\chi-\chi_M)}+\frac{\chi_Q}{\chi_Q-\chi}\right) > 0,$$
$$\frac{d\chi}{d\lambda} = \frac{1}{\lambda H}\left(\frac{\chi_M}{\chi-\chi_M}+\frac{\chi_Q}{\chi_Q-\chi}\right) > 0,$$

where $H = 1/(\chi(1-\chi)) + (\chi_Q-\chi_M)/((\chi-\chi_M)(\chi_Q-\chi)) > 0$ for $\chi \in (\chi_M, \chi_Q)$.

References

[1] Cheng, L., Tao, Z., 1999. The impact of public policies on innovation and imitation: The role of R&D technology in growth models. International Economic Review 40, 187-207.

[2] Cheng, L., Dinopoulos, E., 1996. A multisectoral general equilibrium model of Shumpeterian of growth and fluctuations. Journal of Economic Dynamics and Control 20, 905-923.

[3] Cohen, W., Klepper, S., 1996. Firm size and the nature of innovation within industries: The case of process and product R&D. The Review of Economic Studies 78, 232-243.

[4] Davidson, C., Segerstrom, P., 1998. R&D subsidies and economic growth. RAND Journal of Economics 29, 548-577.

[5] Flamm, K., 1993. Measurement of DRAM prices: Technology and market structure. Price Measurements and Their Uses, 157-197.

[6] Grimm, B., 1998. Price indexes for selected semiconductors, 1974-96. Survey of Current Business 78, 8-24.

[7] Grossman, G., Helpman, E., 1991. Quality ladders in the theory of growth. Review of Economic Studies 58, 43-61.

[8] Haine, S., 2009. A Brief History of the Walkman. TIME Magazine, July 1.

[9] Irwin, D., Klenow, P., 1994. Learning by doing in the semiconductor industry. The Journal of Political Economy 102, 1200-1227.

[10] Klepper, S., Graddy, E., 1990. The evolution of new industries and the determinants of market structure. The RAND Journal of Economics 21, 27-44.

[11] Klepper, S., 1996. Entry, exit, growth, and innovation over the product life cycle. The American Economic Review 86, 562-583.

[12] Peretto, P., 1996. Sunk costs, market structure, and growth. International Economic Review 37, 895-923.

[13] Peretto, P., Connolly, M., 2007. The Manhattan Metaphor. Journal of Economic Growth 12, 329-350.

[14] Segerstrom, P., 1991. Innovation, imitation, and economic growth. Journal of Political Economy 99, 807-827.

[15] Smulders, S., van de Klundert, T., 1995. Imperfect competition, concentration and growth with firm specific R&D. European Economic Review 39, 139-160.